Instructions

- Use black ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- Calculators may be used.
- You must NOT write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for each question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.
International GCSE MATHEMATICS
FORMULAE SHEET – HIGHER TIER

Pythagoras’ Theorem

\[ a^2 + b^2 = c^2 \]

Volume of cone = \( \frac{1}{3} \pi r^2h \)

Curved surface area of cone = \( \pi rl \)

Volume of sphere = \( \frac{4}{3} \pi r^3 \)

Surface area of sphere = \( 4\pi r^2 \)

Theorem

\[ a^2 + b^2 = c^2 \]

\[ \text{adj} = \text{hyp} \times \cos \theta \]
\[ \text{opp} = \text{hyp} \times \sin \theta \]
\[ \text{opp} = \text{adj} \times \tan \theta \]

or
\[ \sin \theta = \frac{\text{opp}}{\text{hyp}} \]
\[ \cos \theta = \frac{\text{adj}}{\text{hyp}} \]
\[ \tan \theta = \frac{\text{opp}}{\text{adj}} \]

Area of triangle

\[ \frac{1}{2} \sin A \cdot b \cdot c \]

Area of a trapezium = \( \frac{1}{2}(a + b)h \)

Volume of prism = area of cross section \( \times \) length

Circumference of circle = \( 2\pi r \)

Area of circle = \( \pi r^2 \)

Volume of cylinder = \( \pi r^2h \)

Curved surface area of cylinder = \( 2\pi rh \)

The Quadratic Equation

The solutions of \( ax^2 + bx + c = 0 \), where \( a \neq 0 \), are given by

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
Answer ALL TWENTY ONE questions.

Write your answers in the spaces provided.

You must write down all stages in your working.

1. (a) Complete the table to show each number written correct to 1 significant figure.

<table>
<thead>
<tr>
<th>Number</th>
<th>42.37</th>
<th>58.92</th>
<th>21.04</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number written correct to 1 significant figure</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(2)

(b) Use the approximations in part (a) to work out an estimate for the value of

\[
\frac{42.37 + 58.92}{21.04}
\]

Show clearly how you obtain your answer.

(2)

(Total for Question 1 is 4 marks)

Do NOT write in this space.
2  \( E = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \),  
\( A = \{1, 3, 5, 7\} \),  
\( B = \{2, 4, 6, 8\} \)

(a) Explain why \( A \cap B = \emptyset \)

\[ x \in E \text{ and } x \notin A \cup B \]

(b) Write down the value of \( x \).

\[ x = \ldots \]  

\( A \cap C = \{3, 7\}, B \cap C = \{8\} \) and \( A \cup B \cup C = E \)

(c) List all the members of \( C \).
The diagram shows a rectangle \(PQRS\).
\(PQ = 14\) cm and \(QR = 9\) cm.
The point \(A\) lies on \(PS\) so that \(PA = 5\) cm.
The point \(B\) lies on \(SR\) so that \(BR = 8\) cm.

(a) Work out the area of triangle \(AQB\).

\[\text{cm}^2\]

(b) Work out the length of \(AQ\).
Give your answer correct to 3 significant figures.

\[\text{cm}\]

(Total for Question 3 is 7 marks)
Freya keeps hens. The table shows information about the number of boxes of eggs she sold in each of 52 weeks.

<table>
<thead>
<tr>
<th>Number of boxes sold in a week</th>
<th>Number of weeks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 4</td>
<td>2</td>
</tr>
<tr>
<td>5 to 9</td>
<td>6</td>
</tr>
<tr>
<td>10 to 14</td>
<td>20</td>
</tr>
<tr>
<td>15 to 19</td>
<td>13</td>
</tr>
<tr>
<td>20 to 24</td>
<td>8</td>
</tr>
<tr>
<td>25 to 29</td>
<td>3</td>
</tr>
</tbody>
</table>

(a) Write down the modal class.

........................................................................

(1)

(b) Work out an estimate for the mean number of boxes of eggs that Freya sold each week. Give your answer correct to 3 significant figures.

........................................................................

(4)
Dan picks at random one of the 52 weeks.

(c) Find the probability that in this week Freya sold at least 15 boxes of eggs.

5  (a) Factorise $7dg - 9de$

(b) Expand and simplify $(x + 2)(x + 5)$

(Total for Question 5 is 4 marks)
6 Solve $3(2z - 5) = 4z + 11$
Show clear algebraic working.

\[ z = \ldots \]

(Total for Question 6 is 3 marks)

7 The table gives some information about the average price of a litre of petrol in England.

<table>
<thead>
<tr>
<th>Average price of a litre of petrol (pence)</th>
<th>January 2007</th>
<th>January 2012</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>87.3</td>
<td>133.3</td>
</tr>
</tbody>
</table>

(a) Work out the percentage increase in the average price of a litre of petrol in England between January 2007 and January 2012.
Give your answer correct to 3 significant figures.

\[ \ldots \% \]

(3)
The average price of a litre of petrol in England increased by 20% from January 2010 to January 2012.

(b) Work out the average price of a litre of petrol in England in January 2010.
   Give your answer in pence, correct to 1 decimal place.

                        pence

(3)

(Total for Question 7 is 6 marks)

Do NOT write in this space.
8 (a) Complete the table of values for \( y = x^2 - 5x + 4 \)

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>

(2)

(b) On the grid, draw the graph of \( y = x^2 - 5x + 4 \) for all values of \( x \) from \( x = 0 \) to \( x = 5 \)

(2)

(Total for Question 8 is 4 marks)
9 A cylinder has diameter 12 cm and length 30 cm.

Work out the curved surface area of the cylinder.
Give your answer correct to 3 significant figures.

\[ \text{Curved surface area} = 2\pi rh \]

\[ = 2\pi \times 6 \times 30 \]

\[ = 360\pi \]

\[ \approx 1131 \text{ cm}^2 \]

(Total for Question 9 is 3 marks)
10 The cumulative frequency graph gives information about the monthly rainfall, in millimetres, in the United Kingdom during 120 months in the years 2001 to 2010.

(a) Use the graph to estimate the number of months for which rainfall was less than 50 mm.

.................................................................................................................

(1)

(b) Use the graph to find an estimate for the median monthly rainfall.

................................................................................................................. mm

(1)

(c) Use the graph to find an estimate for the interquartile range of the monthly rainfall.

................................................................................................................. mm

(2)

(Total for Question 10 is 4 marks)
The functions $f$ and $g$ are defined as

\[ f(x) = \frac{1}{2}x + 4 \]
\[ g(x) = \frac{2x}{x + 1} \]

(a) Work out $f(6)$

..........................................

(1)

(b) Work out $fg(-3)$

..........................................

(2)

c) $g(a) = -2$

Work out the value of $a$.

\[ a = \]  

(2)

d) Express the inverse function $f^{-1}$ in the form $f^{-1}(x) =$

\[ f^{-1}(x) = \]  

(3)

(Total for Question 11 is 8 marks)
In the diagram, $DAPS$ and $CBQR$ are straight lines. $AB$ is parallel to $QP$ and $DC$ is parallel to $RS$.

$AD = 11$ cm, $BC = 5$ cm, $PS = 27.5$ cm and $RS = 42.5$ cm.

Quadrilateral $ABCD$ is similar to quadrilateral $PQRS$.

(a) Find the ratio of the length of $AB$ to the length of $PQ$.

Give your answer in the form $1 : n$

1 : ........................................

(2)

(b) Work out the length of $RQ$.

......................................... cm

(2)

(c) Work out the length of $CD$.

......................................... cm

(2)
The area of quadrilateral $ABCD$ is 54 cm$^2$

(d) Work out the area of quadrilateral $PQRS$.

\[ \text{\phantom{0000} cm}^2 \]  \hspace{1cm} (2)

(Total for Question 12 is 8 marks)

13 Solve the simultaneous equations

\[
3x + 4y = 6 \\
5x + 6y = 11
\]

Show clear algebraic working.

\[ x = \text{\phantom{0000}} \]

\[ y = \text{\phantom{0000}} \]

(Total for Question 13 is 4 marks)
14 (a) \( y = 2x^3 + 3x^2 + 2 \)

Find \( \frac{dy}{dx} \)

(b) The point \( P \) lies on the curve with equation \( y = 2x^3 + 3x^2 + 2 \)

The gradient of the curve at \( P \) is \( -\frac{3}{2} \)

Find the coordinates of \( P \).

\( (..........................,..........................) \)

(Total for Question 14 is 7 marks)
The histogram shows information about the times taken by a telephone call centre to answer incoming calls.

Work out an estimate for the percentage of calls that are answered in less than 40 seconds.

\[ \text{Frequency density} \]

\[ \text{Time (seconds)} \]

\[ 0 \quad 20 \quad 40 \quad 60 \quad 80 \quad 100 \quad 120 \]

\[ 0 \quad 1 \quad 2 \quad 3 \]

\[ \text{(%)} \]

(Total for Question 15 is 3 marks)
16 (a) Expand \((5 + 3\sqrt{2})^2\)

Give your answer in the form \((a + b\sqrt{2})\), where \(a\) and \(b\) are integers.
Show your working clearly.

(b) \((5 + 3\sqrt{2})^2 = p + \frac{q}{\sqrt{8}}\), where \(p\) and \(q\) are integers.
Find the value of \(q\).

\[q = \ldots\]
17 The table shows information about the 40 coins in Karam’s money box.

<table>
<thead>
<tr>
<th>Value of coin (pence)</th>
<th>Bronze coins</th>
<th>Silver coins</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Karam shakes his money box until a coin falls out at random. He does not replace the coin in the money box. Karam shakes his money box again until a second coin falls out at random.

(a) Work out the probability that both the coins that fall out are silver coins.

(b) Work out the probability that the total value of the two coins that fall out is 60 pence or more.

(Total for Question 17 is 5 marks)
The diagram shows a cuboid $ABCDEFGH$.
$AB = 8$ cm, $AF = 6$ cm and $FC = 16$ cm.

(a) Find the length of $BC$.
   Give your answer correct to 3 significant figures.

(b) Find the size of the angle between the line $FC$ and the plane $ABGF$.
   Give your answer correct to 1 decimal place.

(Total for Question 18 is 5 marks)
19 Solve the inequality \( 3x^2 + 5 < 53 \)

(Total for Question 19 is 3 marks)

20 Solve the equation \( \frac{2^{(n^2)}}{2^n \times 2^6} = 1 \)

Show clear algebraic working.

(Total for Question 20 is 3 marks)
$OABC$ is a parallelogram.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$

$P$ is the point on $AB$ such that $\overrightarrow{AP} = \frac{1}{4} \overrightarrow{AB}$.

$Q$ is the point on $OC$ such that $\overrightarrow{OQ} = \frac{2}{3} \overrightarrow{OC}$.

Find, in terms of $\mathbf{a}$ and $\mathbf{c}$, $\overrightarrow{PQ}$.
Give your answer in its simplest form.