Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Core Mathematics C34

Advanced

Monday 26 January 2015 – Afternoon
Time: 2 hours 30 minutes

Paper Reference
WMA02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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1. The curve C has equation

$$y = \frac{3x - 2}{(x - 2)^2}, \quad x \neq 2$$

The point P on C has x coordinate 3

Find an equation of the normal to C at the point P in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

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Question 1 continued

Q1

(Total 6 marks)

P 4 5 0 5 8 A 0 3 4 8

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2. Solve, for $0 \leqslant \theta < 2\pi$,

$$2\cos 2\theta = 5 - 13\sin\theta$$

Give your answers in radians to 3 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

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Question 2 continued

(Total 5 marks)

Q2

Turn over

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3. The function g is defined by

$$g : x \mapsto |8 - 2x|, \qquad x \in \mathbb{R}, \qquad x \geqslant 0$$

(a) Sketch the graph with equation $y = g(x)$, showing the coordinates of the points where the graph cuts or meets the axes.

(3)

(b) Solve the equation

$$|8 - 2x| = x + 5$$

(3)

The function f is defined by

$$f : x \mapsto x^2 - 3x + 1, \qquad x \in \mathbb{R}, \qquad 0 \leqslant x \leqslant 4$$

(c) Find fg(5).

(2)

(d) Find the range of f. You must make your method clear.

(4)

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Question 3 continued

Leave blank

Question 3 continued

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Question 3 continued

(Total 12 marks)

Q3

P 4 5 0 5 8 A 0 9 4 8

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4. Use the substitution $x = 2\sin\theta$ to find the exact value of

$$\int_0^{\sqrt{3}} \frac{1}{(4-x^2)^{\frac{3}{2}}}\,\mathrm{d}x$$

(7)

Leave blank

Question 4 continued

(Total 7 marks)

Q4

P 4 5 0 5 8 A 0 1 1 4 8

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5. (a) Use the binomial expansion, in ascending powers of x, of $\frac{1}{\sqrt{(1-2x)}}$ to show that

$$\frac{2+3x}{\sqrt{(1-2x)}} \approx 2 + 5x + 6x^2, \qquad |x| < 0.5$$

(4)

(b) Substitute $x = \frac{1}{20}$ into

$$\frac{2+3x}{\sqrt{(1-2x)}} = 2 + 5x + 6x^2$$

to obtain an approximation to $\sqrt{10}$

Give your answer as a fraction in its simplest form.

(3)

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Question 5 continued

P 4 5 0 5 8 A 0 1 3 4 8

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Question 5 continued

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Question 5 continued

Q5

(Total 7 marks)

P 4 5 0 5 8 A 0 1 5 4 8

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6. (i) Given $x = \tan^2 4y$, $0 < y < \dfrac{\pi}{8}$, find $\dfrac{dy}{dx}$ as a function of x.

Write your answer in the form $\dfrac{1}{A(x^p + x^q)}$, where A, p and q are constants to be found.

(5)

(ii) The volume V of a cube is increasing at a constant rate of 2 cm^3 s^{-1}. Find the rate at which the length of the edge of the cube is increasing when the volume of the cube is 64 cm^3.

(5)

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Question 6 continued

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Question 6 continued

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Question 6 continued

Q6

(Total 10 marks)

P 4 5 0 5 8 A 0 1 9 4 8

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7. (a) Given that

$$2\cos(x + 30)^\circ = \sin(x - 30)^\circ$$

without using a calculator, show that

$$\tan x^\circ = 3\sqrt{3} - 4$$

(5)

(b) Hence or otherwise solve, for $0 \leqslant \theta < 180$,

$$2\cos(2\theta + 40)^\circ = \sin(2\theta - 20)^\circ$$

Give your answers to one decimal place.

(4)

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 9 marks)

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8.

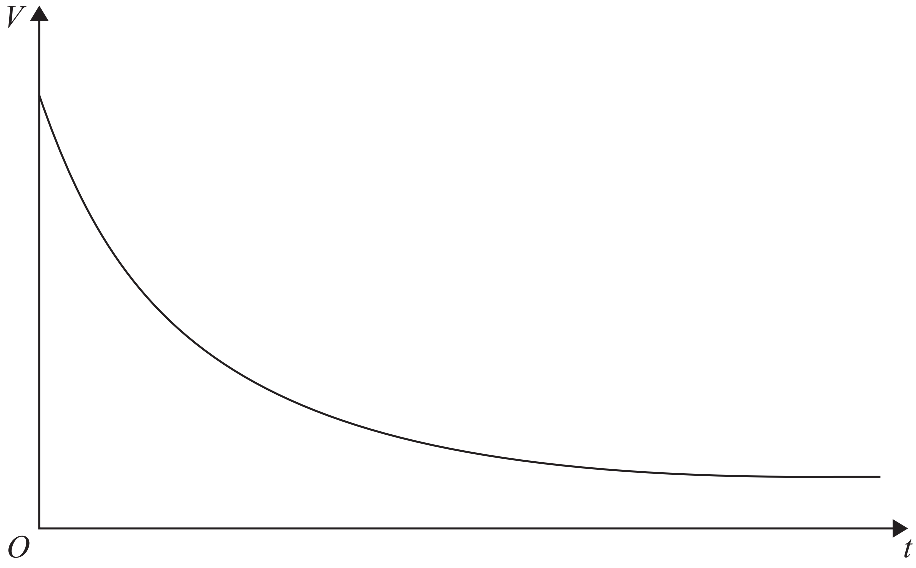

Figure 1

The value of Lin's car is modelled by the formula

$$V = 18\,000e^{-0.2t} + 4000e^{-0.1t} + 1000, \quad t \geqslant 0$$

where the value of the car is V pounds when the age of the car is t years.

A sketch of t against V is shown in Figure 1.

(a) State the range of V.

(2)

According to this model,

(b) find the rate at which the value of the car is decreasing when $t = 10$
Give your answer in pounds per year.

(3)

(c) Calculate the exact value of t when $V = 15000$

(4)

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

(Total 9 marks)

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9.

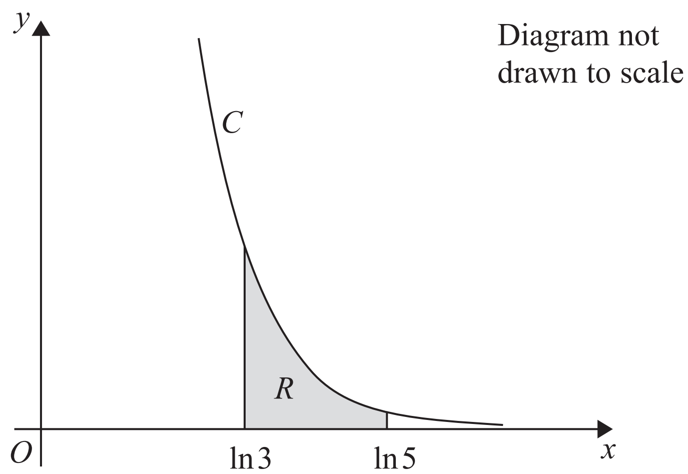

Figure 2

The curve C has parametric equations

$$x = \ln(t+2), \quad y = \frac{4}{t^2} \qquad t > 0$$

The finite region R, shown shaded in Figure 2, is bounded by the curve C, the x-axis and the lines with equations $x = \ln 3$ and $x = \ln 5$

(a) Show that the area of R is given by the integral

$$\int_1^3 \frac{4}{t^2(t+2)}\,\mathrm{d}t$$

(3)

(b) Hence find an exact value for the area of R.

Write your answer in the form $(a + \ln b)$, where a and b are rational numbers.

(7)

(c) Find a cartesian equation of the curve C in the form $y = \mathrm{f}(x)$.

(2)

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Question 9 continued

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Question 9 continued

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Question 9 continued

Q9

(Total 12 marks)

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10.

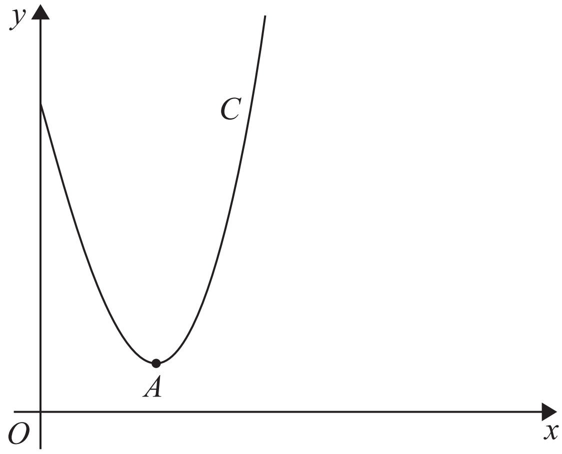

Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 4, \quad x > 0$$

Point A is the minimum turning point on the curve.

(a) Show, by using calculus, that the x coordinate of point A is a solution of

$$x = \frac{6}{1 + \ln(x^2)}$$

(5)

(b) Starting with $x_0 = 2.27$, use the iteration

$$x_{n+1} = \frac{6}{1 + \ln(x_n^{\ 2})}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(c) Use your answer to part (b) to deduce the coordinates of point A to one decimal place.

(2)

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Question 10 continued

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Question 10 continued

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Question 10 continued

(Total 10 marks)

Q10

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11. With respect to a fixed origin O the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 14 \\ -6 \\ -13 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} \qquad l_2: \mathbf{r} = \begin{pmatrix} p \\ -7 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 1 \end{pmatrix}$$

where λ and μ are scalar parameters and p and q are constants.

Given that l_1 and l_2 are perpendicular,

(a) show that $q = 3$

(2)

Given further that l_1 and l_2 intersect at point X,

find

(b) the value of p,

(5)

(c) the coordinates of X.

(2)

The point A lies on l_1 and has position vector $\begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$

Given that point B also lies on l_1 and that $AB = 2AX$

(d) find the two possible position vectors of B.

(3)

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Question 11 continued

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Question 11 continued

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Question 11 continued

Q11

(Total 12 marks)

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12.

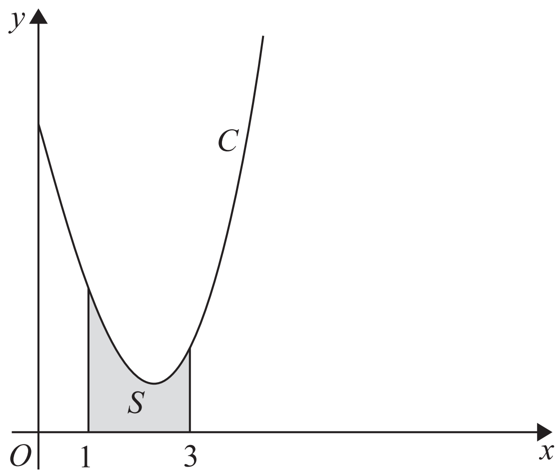

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 4, \quad x > 0$$

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the x-axis and the lines with equations $x = 1$ and $x = 3$

(a) Complete the table below with the value of y corresponding to $x = 2$. Give your answer to 4 decimal places.

x	1	1.5	2	2.5	3
y	2	1.3041		0.9089	1.2958

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of S, giving your answer to 3 decimal places.

(3)

(c) Use calculus to find the exact area of S.

Give your answer in the form $\frac{a}{b} + \ln c$, where a, b and c are integers.

(6)

(d) Hence calculate the percentage error in using your answer to part (b) to estimate the area of S. Give your answer to one decimal place.

(2)

(e) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of S.

(1)

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Question 12 continued

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Question 12 continued

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Question 12 continued

Q12

(Total 13 marks)

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13. (a) Express $10\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$

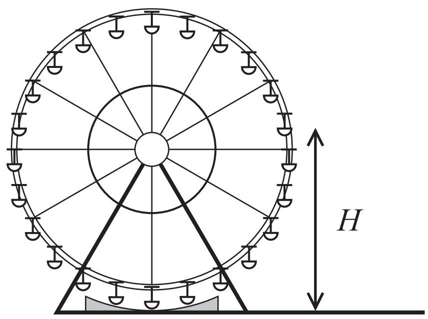
Give the exact value of R and give the value of α to 2 decimal places.

(3)

Alana models the height above the ground of a passenger on a Ferris wheel by the equation

$$H = 12 - 10\cos(30t)° + 3\sin(30t)°$$

where the height of the passenger above the ground is H metres at time t minutes after the wheel starts turning.



(b) Calculate

(i) the maximum value of H predicted by this model,

(ii) the value of t when this maximum first occurs.

Give each answer to 2 decimal places.

(4)

(c) Calculate the value of t when the passenger is 18 m above the ground for the first time.

Give your answer to 2 decimal places.

(4)

(d) Determine the time taken for the Ferris wheel to complete two revolutions.

(2)

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Question 13 continued

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Question 13 continued

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Question 13 continued

Q13

(Total 13 marks)

TOTAL FOR PAPER: 125 MARKS

END

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