Edexcel GCE
Core Mathematics C3
Advanced
Monday 16 June 2014 – Morning
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper. Answer ALL the questions.
You must write your answer for each question in the space following the question. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided. Full marks may be obtained for answers to ALL questions. The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled. You should show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.
1. Express

\[
\frac{3}{2x + 3} - \frac{1}{2x - 3} + \frac{6}{4x^2 - 9}
\]

as a single fraction in its simplest form. (4)
2. A curve $C$ has equation $y = e^{4x} + x^4 + 8x + 5$

(a) Show that the $x$ coordinate of any turning point of $C$ satisfies the equation

$$x^3 = -2 - e^{4x}$$

(b) On the axes given on page 5, sketch, on a single diagram, the curves with equations

(i) $y = x^3$,  
(ii) $y = -2 - e^{4x}$

On your diagram give the coordinates of the points where each curve crosses the $y$-axis and state the equation of any asymptotes.

(c) Explain how your diagram illustrates that the equation $x^3 = -2 - e^{4x}$ has only one root.

The iteration formula

$$x_{n+1} = (-2 - e^{4x})^{\frac{1}{3}}, \quad x_0 = -1$$

can be used to find an approximate value for this root.

(d) Calculate the values of $x_1$ and $x_2$, giving your answers to 5 decimal places.

(e) Hence deduce the coordinates, to 2 decimal places, of the turning point of the curve $C$. 

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Question 2 continued

\[ y \]

\[ x \]

\[ O \]
Question 2 continued
3. (i) (a) Show that \(2 \tan x - \cot x = 5 \cosec x\) may be written in the form

\[a \cos^2 x + b \cos x + c = 0\]

stating the values of the constants \(a\), \(b\) and \(c\).

(b) Hence solve, for \(0 < x < 2\pi\), the equation

\[2 \tan x - \cot x = 5 \cosec x\]

giving your answers to 3 significant figures.

(ii) Show that

\[\tan \theta + \cot \theta = \lambda \ \cosec 2\theta, \quad \theta \neq \frac{n\pi}{2}, \quad n \in \mathbb{Z}\]

stating the value of the constant \(\lambda\).
Question 3 continued
4. (i) Given that
\[ x = \sec^2 2y, \quad 0 < y < \frac{\pi}{4} \]
show that
\[ \frac{dy}{dx} = \frac{1}{4x\sqrt{x - 1}} \]  \hspace{1cm} (4)

(ii) Given that
\[ y = (x^2 + x^3)\ln 2x \]
find the exact value of \( \frac{dy}{dx} \) at \( x = \frac{e}{2} \), giving your answer in its simplest form. \hspace{1cm} (5)

(iii) Given that
\[ f(x) = \frac{3\cos x}{(x + 1)^{\frac{1}{3}}}, \quad x \neq -1 \]
show that
\[ f'(x) = \frac{g(x)}{(x + 1)^{\frac{4}{3}}}, \quad x \neq -1 \]
where \( g(x) \) is an expression to be found. \hspace{1cm} (3)
Question 4 continued

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Question 4 continued
5. (a) Sketch the graph with equation

\[ y = |4x - 3| \]

stating the coordinates of any points where the graph cuts or meets the axes. (2)

Find the complete set of values of \( x \) for which

(b) \[ |4x - 3| > 2 - 2x \] (4)

(c) \[ |4x - 3| > \frac{3}{2} - 2x \] (2)
Question 5 continued
Question 5 continued
Question 5 continued

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(Total 8 marks)
The function $f$ is defined by

$$f : x \rightarrow e^{2x} + k^2, \quad x \in \mathbb{R}, \quad k \text{ is a positive constant.}$$

(a) State the range of $f$. (1)

(b) Find $f^{-1}$ and state its domain. (3)

The function $g$ is defined by

$$g : x \rightarrow \ln(2x), \quad x > 0$$

(c) Solve the equation

$$g(x) + g(x^3) + g(x^3) = 6$$

giving your answer in its simplest form. (4)

(d) Find $fg(x)$, giving your answer in its simplest form. (2)

(e) Find, in terms of the constant $k$, the solution of the equation

$$fg(x) = 2k^2$$

(2)
Question 6 continued
Question 6 continued
Question 6 continued

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(Total 12 marks)
7.

Figure 1

Figure 1 shows the curve \( C \), with equation \( y = 6 \cos x + 2.5 \sin x \) for \( 0 \leq x \leq 2\pi \)

(a) Express \( 6 \cos x + 2.5 \sin x \) in the form \( R \cos(x - \alpha) \), where \( R \) and \( \alpha \) are constants

with \( R > 0 \) and \( 0 < \alpha < \frac{\pi}{2} \). Give your value of \( \alpha \) to 3 decimal places.

(b) Find the coordinates of the points on the graph where the curve \( C \) crosses the coordinate axes.

A student records the number of hours of daylight each Sunday throughout the year. She starts on the last Sunday in May with a recording of 18 hours, and continues until her final recording 52 weeks later.

She models her results with the continuous function given by

\[ H = 12 + 6 \cos \left( \frac{2\pi t}{52} \right) + 2.5 \sin \left( \frac{2\pi t}{52} \right), \quad 0 \leq t \leq 52 \]

where \( H \) is the number of hours of daylight and \( t \) is the number of weeks since her first recording.

Use this function to find

(c) the maximum and minimum values of \( H \) predicted by the model,

(d) the values for \( t \) when \( H = 16 \), giving your answers to the nearest whole number.

[You must show your working. Answers based entirely on graphical or numerical methods are not acceptable.]
Question 7 continued
Question 7 continued

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Question 7 continued