

Mark Scheme (Results) January 2010

GCE

Core Mathematics C3 (6665)

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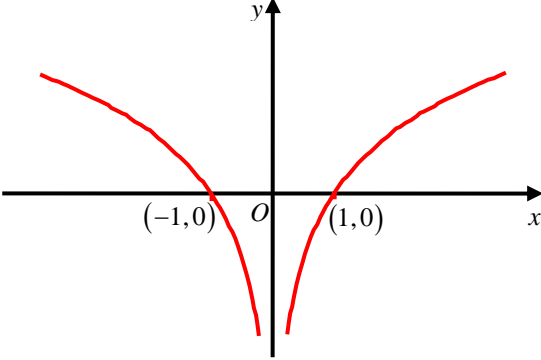
Question Number	Scheme	Marks
Q1	$\frac{x+1}{3x^2-3} - \frac{1}{3x+1}$ $= \frac{x+1}{3(x^2-1)} - \frac{1}{3x+1}$ $= \frac{x+1}{3(x+1)(x-1)} - \frac{1}{3x+1}$ $= \frac{1}{3(x-1)} - \frac{1}{3x+1}$ $= \frac{3x+1-3(x-1)}{3(x-1)(3x+1)}$ <p>or</p> $\frac{3x+1}{3(x-1)(3x+1)} - \frac{3(x-1)}{3(x-1)(3x+1)}$ $= \frac{4}{3(x-1)(3x+1)}$	<p style="text-align: right;">Award below</p> <p>$x^2 - 1 \rightarrow (x+1)(x-1)$ or $3x^2 - 3 \rightarrow (x+1)(3x-3)$ or $3x^2 - 3 \rightarrow (3x+3)(x-1)$ seen or implied anywhere in candidate's working.</p> <p style="text-align: right;">Attempt to combine. M1</p> <p style="text-align: right;">Correct result. A1</p> <p style="text-align: right;">Decide to award M1 here!! M1</p> <p style="text-align: right;">Either $\frac{4}{3(x-1)(3x+1)}$ or $\frac{\frac{4}{3}}{(x-1)(3x+1)}$ or $\frac{4}{(3x-3)(3x+1)}$ or $\frac{4}{9x^2-6x-3}$ A1 aef</p> <p style="text-align: right;">[4]</p>

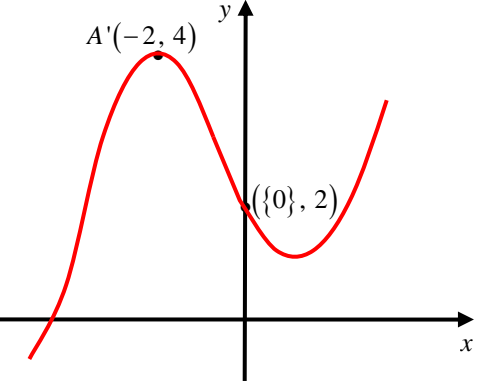

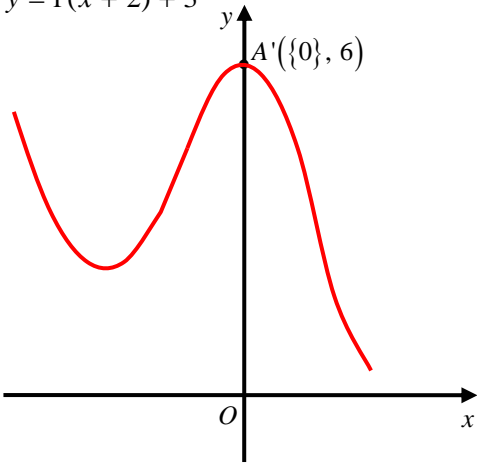
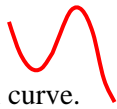
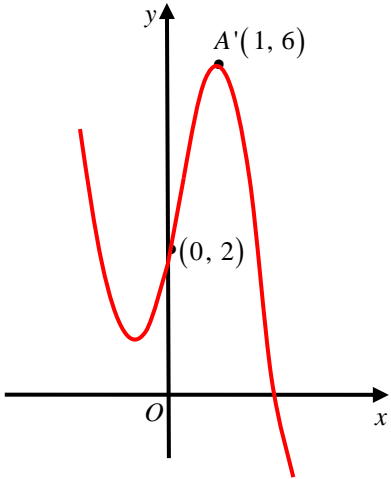

Question Number	Scheme	Marks
<p>Q2</p> <p>(a)</p> <p>(b)</p> <p>(c)</p>	<p>$f(x) = x^3 + 2x^2 - 3x - 11$</p> <p>$f(x) = 0 \Rightarrow x^3 + 2x^2 - 3x - 11 = 0$ $\Rightarrow x^2(x + 2) - 3x - 11 = 0$</p> <p>$\Rightarrow x^2(x + 2) = 3x + 11$ $\Rightarrow x^2 = \frac{3x + 11}{x + 2}$ $\Rightarrow x = \sqrt{\left(\frac{3x + 11}{x + 2}\right)}$</p> <p>Iterative formula: $x_{n+1} = \sqrt{\left(\frac{3x_n + 11}{x_n + 2}\right)}$, $x_1 = 0$</p> <p>$x_2 = \sqrt{\left(\frac{3(0) + 11}{(0) + 2}\right)}$</p> <p>$x_2 = 2.34520788\dots$ $x_3 = 2.037324945\dots$ $x_4 = 2.058748112\dots$</p> <p>Let $f(x) = x^3 + 2x^2 - 3x - 11 = 0$</p> <p>$f(2.0565) = -0.013781637\dots$ $f(2.0575) = 0.0041401094\dots$ Sign change (and $f(x)$ is continuous) therefore a root α is such that $\alpha \in (2.0565, 2.0575) \Rightarrow \alpha = 2.057$ (3 dp)</p>	<p>Sets $f(x) = 0$ (can be implied) and takes out a factor of x^2 from $x^3 + 2x^2$, or x from $x^3 + 2x$ (slip).</p> <p>then rearranges to give the quoted result on the question paper.</p> <p>An attempt to substitute $x_1 = 0$ into the iterative formula. Can be implied by $x_2 = \sqrt{5.5}$ or 2.35 or awrt 2.345</p> <p>Both $x_2 =$ awrt 2.345 and $x_3 =$ awrt 2.037 $x_4 =$ awrt 2.059</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px auto;"> Choose suitable interval for x, e.g. [2.0565, 2.0575] or tighter </div> <p>any one value awrt 1 sf</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px auto;"> both values correct awrt 1sf, sign change and conclusion </div> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px auto;"> As a minimum, both values must be correct to 1 sf, candidate states "change of sign, hence root". </div> <p>[8]</p>

Question Number	Scheme	Marks
Q3 (a)	$5 \cos x - 3 \sin x = R \cos(x + \alpha), \quad R > 0, \quad 0 < x < \frac{\pi}{2}$ $5 \cos x - 3 \sin x = R \cos x \cos \alpha - R \sin x \sin \alpha$ <p>Equate $\cos x$: $5 = R \cos \alpha$ Equate $\sin x$: $3 = R \sin \alpha$</p> $R = \sqrt{5^2 + 3^2}; = \sqrt{34} \quad \{= 5.83095.. \}$ $\tan \alpha = \frac{3}{5} \Rightarrow \alpha = 0.5404195003...^{\circ}$ <p>Hence, $5 \cos x - 3 \sin x = \sqrt{34} \cos(x + 0.5404)$</p>	<p>M1; A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>(4)</p>
(b)	$5 \cos x - 3 \sin x = 4$ $\sqrt{34} \cos(x + 0.5404) = 4$ $\cos(x + 0.5404) = \frac{4}{\sqrt{34}} \quad \{= 0.68599... \}$ $(x + 0.5404) = 0.814826916...^{\circ}$ $x = 0.2744...^{\circ}$ $(x + 0.5404) = 2\pi - 0.814826916...^{\circ} \quad \{= 5.468358...^{\circ} \}$ $x = 4.9279...^{\circ}$ <p>Hence, $x = \{0.27, 4.93\}$</p>	<p>M1</p> <p>M1 A1</p> <p>ddM1 A1</p> <p>(5)</p> <p>[9]</p>

Part (b): If there are any EXTRA solutions inside the range $0 \leq x < 2\pi$, then withhold the final accuracy mark if the candidate would otherwise score all 5 marks. Also ignore EXTRA solutions outside the range $0 \leq x < 2\pi$.

Question Number	Scheme	Marks
Q4 (i)	$y = \frac{\ln(x^2 + 1)}{x}$ $u = \ln(x^2 + 1) \Rightarrow \frac{du}{dx} = \frac{2x}{x^2 + 1}$ <p>Apply quotient rule: $\left\{ \begin{array}{l} u = \ln(x^2 + 1) \quad v = x \\ \frac{du}{dx} = \frac{2x}{x^2 + 1} \quad \frac{dv}{dx} = 1 \end{array} \right\}$</p> $\frac{dy}{dx} = \frac{\left(\frac{2x}{x^2 + 1}\right)(x) - \ln(x^2 + 1)}{x^2}$ $\left\{ \frac{dy}{dx} = \frac{2}{(x^2 + 1)} - \frac{1}{x^2} \ln(x^2 + 1) \right\}$	<p>$\ln(x^2 + 1) \rightarrow \frac{\text{something}}{x^2 + 1}$ M1</p> <p>$\ln(x^2 + 1) \rightarrow \frac{2x}{x^2 + 1}$ A1</p> <p>Applying $\frac{xu' - \ln(x^2 + 1)v'}{x^2}$ correctly. M1</p> <p>Correct differentiation with correct bracketing but allow recovery. A1</p> <p>{Ignore subsequent working.}</p>
(ii)	$x = \tan y$ $\frac{dx}{dy} = \sec^2 y$ $\frac{dy}{dx} = \frac{1}{\sec^2 y} \{ = \cos^2 y \}$ $\frac{dy}{dx} = \frac{1}{1 + \tan^2 y}$ <p>Hence, $\frac{dy}{dx} = \frac{1}{1 + x^2}$, (as required)</p>	<p>$\tan y \rightarrow \sec^2 y$ or an attempt to differentiate $\frac{\sin y}{\cos y}$ using either the quotient rule or product rule. M1*</p> <p>$\frac{dx}{dy} = \sec^2 y$ A1</p> <p>Finding $\frac{dy}{dx}$ by reciprocating $\frac{dx}{dy}$. dM1*</p> <p>For writing down or applying the identity $\sec^2 y = 1 + \tan^2 y$, which must be applied/stated completely in y. dM1*</p> <p>For the correct proof, leading on from the previous line of working. A1 AG</p>
		<p>(4)</p> <p>(5)</p> <p>[9]</p>

Question Number	Scheme	Marks
Q5	<p data-bbox="225 342 325 376">$y = \ln x$</p>  <p data-bbox="906 421 1385 488">Right-hand branch in quadrants 4 and 1. Correct shape.</p> <p data-bbox="922 555 1385 622">Left-hand branch in quadrants 2 and 3. Correct shape.</p> <p data-bbox="962 678 1385 757">Completely correct sketch and both $(-1, \{0\})$ and $(1, \{0\})$</p>	<p data-bbox="1409 432 1441 465">B1</p> <p data-bbox="1409 566 1441 600">B1</p> <p data-bbox="1409 701 1441 734">B1</p> <p data-bbox="1505 790 1536 824">(3)</p> <p data-bbox="1505 857 1536 891">[3]</p>

Question Number	Scheme	Marks
Q6 (i)	<p>$y = f(-x) + 1$</p> 	<p>Shape of </p> <p>and must have a maximum in quadrant 2 and a minimum in quadrant 1 or on the positive y-axis. B1</p> <p>Either $(\{0\}, 2)$ or $A'(-2, 4)$ B1</p> <p>Both $(\{0\}, 2)$ and $A'(-2, 4)$ B1</p> <p>(3)</p>
(ii)	<p>$y = f(x + 2) + 3$</p> 	<p>Any translation of the original curve. </p> <p>The translated maximum has either x-coordinate of 0 (can be implied) or y-coordinate of 6. B1</p> <p>The translated curve has maximum $(\{0\}, 6)$ and is in the correct position on the Cartesian axes. B1</p> <p>(3)</p>
(iii)	<p>$y = 2f(2x)$</p> 	<p>Shape of </p> <p>with a minimum in quadrant 2 and a maximum in quadrant 1. B1</p> <p>Either $(\{0\}, 2)$ or $A'(1, 6)$ B1</p> <p>Both $(\{0\}, 2)$ and $A'(1, 6)$ B1</p> <p>(3)</p> <p>[9]</p>

Question Number	Scheme	Marks
Q7 (a)	$y = \sec x = \frac{1}{\cos x} = (\cos x)^{-1}$ $\frac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$ $\frac{dy}{dx} = \left\{ \frac{\sin x}{\cos^2 x} \right\} = \left(\frac{1}{\cos x} \right) \left(\frac{\sin x}{\cos x} \right) = \underline{\underline{\sec x \tan x}}$	$\frac{dy}{dx} = \pm ((\cos x)^{-2}(\sin x))$ <p>M1</p> $-1(\cos x)^{-2}(-\sin x) \text{ or } (\cos x)^{-2}(\sin x)$ <p>A1</p> <p>Convincing proof. Must see both <u>underlined steps.</u></p> <p>A1 AG</p> <p>(3)</p>
(b)	$y = e^{2x} \sec 3x$ $\left\{ \begin{array}{l} u = e^{2x} \quad v = \sec 3x \\ \frac{du}{dx} = 2e^{2x} \quad \frac{dv}{dx} = 3 \sec 3x \tan 3x \end{array} \right\}$ $\frac{dy}{dx} = 2e^{2x} \sec 3x + 3e^{2x} \sec 3x \tan 3x$	<div style="border: 1px solid black; padding: 5px; display: inline-block; margin-right: 20px;">Seen or implied</div> <p>Either $e^{2x} \rightarrow 2e^{2x}$ or $\sec 3x \rightarrow 3 \sec 3x \tan 3x$</p> <p>Both $e^{2x} \rightarrow 2e^{2x}$ and $\sec 3x \rightarrow 3 \sec 3x \tan 3x$</p> <p>M1</p> <p>A1</p> <p>Applies $vu' + uv'$ correctly for their u, u', v, v'</p> <p>M1</p> $2e^{2x} \sec 3x + 3e^{2x} \sec 3x \tan 3x$ <p>A1 isw</p> <p>(4)</p>
(c)	<p>Turning point $\Rightarrow \frac{dy}{dx} = 0$</p> <p>Hence, $e^{2x} \sec 3x(2 + 3 \tan 3x) = 0$</p> <p>{Note $e^{2x} \neq 0$, $\sec 3x \neq 0$, so $2 + 3 \tan 3x = 0$, }</p> <p>giving $\tan 3x = -\frac{2}{3}$</p> <p>$\Rightarrow 3x = -0.58800 \Rightarrow x = \{a\} = -0.19600\dots$</p> <p>Hence, $y = \{b\} = e^{2(-0.196)} \sec(3 \times -0.196)$</p> <p style="text-align: center;">$= 0.812093\dots = 0.812$ (3sf)</p>	<p>Sets their $\frac{dy}{dx} = 0$ and factorises (or cancels) out at least e^{2x} from at least two terms.</p> <p>M1</p> <p>$\tan 3x = \pm k$; $k \neq 0$</p> <p>M1</p> <p>Either awrt -0.196° or awrt -11.2°</p> <p>A1</p> <p>0.812</p> <p>A1 cao</p> <p>(4)</p>

[11]

Part (c): If there are any EXTRA solutions for x (or a) inside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$ or ANY EXTRA solutions for y (or b), (for these values of x) then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$.

Question Number	Scheme	Marks
Q8	<p>$\operatorname{cosec}^2 2x - \cot 2x = 1, \text{ (eqn *) } 0 \leq x \leq 180^\circ$</p> <p>Using $\operatorname{cosec}^2 2x = 1 + \cot^2 2x$ gives</p> <p>$1 + \cot^2 2x - \cot 2x = 1$</p> <p>$\cot^2 2x - \cot 2x = 0$ or $\cot^2 2x = \cot 2x$</p> <p>$\cot 2x(\cot 2x - 1) = 0$ or $\cot 2x = 1$</p> <p>$\cot 2x = 0$ or $\cot 2x = 1$</p> <p>$\cot 2x = 0 \Rightarrow (\tan 2x \rightarrow \infty) \Rightarrow 2x = 90, 270$</p> <p>$\Rightarrow x = 45, 135$</p> <p>$\cot 2x = 1 \Rightarrow \tan 2x = 1 \Rightarrow 2x = 45, 225$</p> <p>$\Rightarrow x = 22.5, 112.5$</p> <p>Overall, $x = \{22.5, 45, 112.5, 135\}$</p>	<p>Writing down or using $\operatorname{cosec}^2 2x = \pm 1 \pm \cot^2 2x$ or $\operatorname{cosec}^2 \theta = \pm 1 \pm \cot^2 \theta$.</p> <p>For either $\frac{\cot^2 2x - \cot 2x}{\cot^2 2x} = 0$ or $\cot^2 2x = \cot 2x$</p> <p>Attempt to factorise or solve a quadratic (See rules for factorising quadratics) or cancelling out $\cot 2x$ from both sides.</p> <p>Both $\cot 2x = 0$ and $\cot 2x = 1$.</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> Candidate attempts to divide at least one of their principal angles by 2. This will be usually implied by seeing $x = 22.5$ resulting from $\cot 2x = 1$. </div> <p>Both $x = 22.5$ and $x = 112.5$ Both $x = 45$ and $x = 135$</p>
		<p>M1</p> <p>A1</p> <p>dM1</p> <p>A1</p> <p>ddM1</p> <p>A1</p> <p>B1</p>
		[7]

If there are any EXTRA solutions inside the range $0 \leq x \leq 180^\circ$ and the candidate would otherwise score FULL MARKS then withhold the final accuracy mark (the sixth mark in this question). Also ignore EXTRA solutions outside the range $0 \leq x \leq 180^\circ$.

Question Number	Scheme	Marks
Q9 (i)(a)	$\ln(3x - 7) = 5$ $e^{\ln(3x - 7)} = e^5$ $3x - 7 = e^5 \Rightarrow x = \frac{e^5 + 7}{3} \{= 51.804...\}$	<p>Takes e of both sides of the equation. This can be implied by $3x - 7 = e^5$.</p> <p>Then rearranges to make x the subject.</p> <p><i>Exact answer</i> of $\frac{e^5 + 7}{3}$.</p> <p>M1 dM1 A1 (3)</p>
(b)	$3^x e^{7x+2} = 15$ $\ln(3^x e^{7x+2}) = \ln 15$ $\ln 3^x + \ln e^{7x+2} = \ln 15$ $x \ln 3 + 7x + 2 = \ln 15$ $x(\ln 3 + 7) = -2 + \ln 15$ $x = \frac{-2 + \ln 15}{7 + \ln 3} \{= 0.0874...\}$	<p>Takes ln (or logs) of both sides of the equation.</p> <p>Applies the addition law of logarithms.</p> $x \ln 3 + 7x + 2 = \ln 15$ <p>Factorising out at least two x terms on one side and collecting number terms on the other side.</p> <p><i>Exact answer</i> of $\frac{-2 + \ln 15}{7 + \ln 3}$</p> <p>M1 M1 A1 oe ddM1 A1 oe (5)</p>
(ii) (a)	$f(x) = e^{2x} + 3, x \in \mathbb{R}$ $y = e^{2x} + 3 \Rightarrow y - 3 = e^{2x}$ $\Rightarrow \ln(y - 3) = 2x$ $\Rightarrow \frac{1}{2} \ln(y - 3) = x$ <p>Hence $f^{-1}(x) = \frac{1}{2} \ln(x - 3)$</p> <p>$f^{-1}(x)$: Domain: $x > 3$ or $(3, \infty)$</p>	<p>Attempt to make x (or swapped y) the subject</p> <p>Makes e^{2x} the subject and takes ln of both sides</p> $\frac{1}{2} \ln(x - 3) \text{ or } \ln \sqrt{(x - 3)}$ <p><u>A1</u> <u>cao</u></p> <p>Either $x > 3$ or $(3, \infty)$ or <u>Domain</u> > 3.</p> <p>B1 (4)</p>
(b)	$g(x) = \ln(x - 1), x \in \mathbb{R}, x > 1$ $fg(x) = e^{2 \ln(x-1)} + 3 \{= (x - 1)^2 + 3\}$ <p>$fg(x)$: Range: $y > 3$ or $(3, \infty)$</p>	<p>An attempt to put function g into function f.</p> $e^{2 \ln(x-1)} + 3 \text{ or } (x - 1)^2 + 3 \text{ or } x^2 - 2x + 4.$ <p>Either $y > 3$ or $(3, \infty)$ or <u>Range</u> > 3 or <u>$fg(x) > 3$</u>.</p> <p>M1 A1 isw B1 (3)</p>

[15]

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